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National DRYLAND SALINITY Program

COOPERATIVE RESEARCH CENTRE FOR CATCHMENT HYDROLOGY

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Impacts of irrigation and dryland development on groundwater discharge to rivers: A unit response approach to cumulative impacts analysis

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Abstract: The influence of irrigation and dryland development has increased groundwater discharge and salt load to adjacent rivers. This paper describes further development of a unit response approach to assess the effect and timing of changes to recharge on groundwater discharge to a river. The unit response approach uses a linearised Boussinesq equation to develop an effectively one dimensional unit response function for a change in recharge to an aquifer some distance from the river. The Glover and Balmer solution for stream depletion is extended from a point to an area, and the effects of both a sloping and a bounded aquifer are considered. This approach can provide approximate lead-in and lag times for the impacts of recharge change on groundwater discharge and salt load to rivers. This approach has been applied to estimate the effects of the development of the Bookpurnong Irrigation Area (Loxton, South Australia).

Key Assumptions

The following paper describes the development and application of a unit response approach to assess the effect and timing of changes to groundwater recharge on groundwater discharge to a river some distance away. There are several important assumptions which limit the applicability of this approach. The main assumptions are as follows:

- the aquifer has a horizontal base (with the exception of specific sloping case in Section 2.5), and its diffusivity is uniform throughout;
- there are only small fluctuations about a mean groundwater level, which implicitly means that the saturated aquifer thickness does not change with time;
- the stream water surface is fixed and constant, and equal to the aquifer head at the river;
- the applied recharge change is uniform across the discrete area of change;
- a groundwater salinity at the river/floodplain must be assumed in order to convert the groundwater flux into a salt load.

Acknowledgments

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1. Introduction

Salinisation of river systems is a serious and increasing problem in southern Australia [*Jolly et al.*, 2001]. Irrigation areas were initially considered to be the major driver of rising stream salinity in south-eastern Australia [*MDBMC*, 1987]. Enhanced recharge under intensive irrigated agriculture developments adjacent to the Murray River in South Australia over the last 100 years has resulted in increased discharge of saline groundwater into the river, resulting in rising river salt loads and salinities.

More recently, the enormous future impact of dryland farming with regard to increased salinity has been acknowledged [*MDBMC*, 1999]. The clearing of deep-rooted perennial native vegetation and its replacement with annual crops and pastures has altered the natural hydrological balance across large areas. The enhanced recharge afforded by this hydrological change in dryland areas has the capacity to increase groundwater discharge. Because of the large salt stores present in many parts of Australia, this discharge expresses itself as increased land salinisation and increased river salinity. For example, increases in groundwater hydraulic heads which have been observed in parts of the Lower Murray region of South Australia will begin to impact on the Murray River over the next 50 to 100 years, leading to future increases in groundwater discharge and hence river salinity [*Allison et al.*, 1990].

Being able to estimate the timing of the groundwater response to changes in recharge, and the effect of these changes on groundwater discharge to a river, is an important step for assessing management implications of new irrigation developments, groundwater pumping schemes, revegetation strategies on stream flow (and hence on stream salt load).

Analytical solutions can help provide answers to these questions, particularly in areas with limited measured data. *Theis* [1941] published an integral solution for the effect of well pumping on stream flow depletion, which was later solved using the complementary error function by *Glover and Balmer* [1954]. *Jenkins* [1977] then published a manual of look-up curves for this solution, which led to more widespread usage. More recently, *Sophocleous et al.* [1999] questioned the validity of several assumptions, and evaluated the solution against MODFLOW [*McDonald and Harbaugh*, 1988]. *Hunt* [1995], *Zlotnik and Huang* [1999], and others have also examined the effects of assumptions such as partially penetrating streams, and streambed clogging. To make the *Glover and Balmer* [1954] solution more relevant to the situation of increased recharge, it has been extended to allow recharge to occur across a discrete area, rather than at a point (well).

A linearised Boussinesq equation has been used (for example see *Gelhar and Wilson* [1974]) to infer a hydraulic timescale over which changes in input to the system (groundwater recharge) are reflected in changes in output (groundwater discharge to a river) [*Manga*, 1999]. An effectively one dimensional analytical expression for a unit response function has been calculated using this model of the two dimensional linearised Boussinesq equation. This function gives the change in groundwater discharge at the stream caused by a unit change in point-scale recharge. This input can be in the form of a pulse or a step change, and can be used for both sources (irrigation or dryland development) or sinks (groundwater pumping). An important groundwater system assumption is that any changes are small relative to the overall storage. Because of this, the individual changes may be summed/combined to calculate the overall temporal stream response to more spatially distributed recharge.

The first part of the paper describes the development of an analytical solution for predicting the effect of recharge change across an area on groundwater flux to rivers. The second part shows an application of the approach to the Bookpurnong Irrigation Area (River Murray, South Australia) as an example.

2. Theoretical Description

This approach describes the influence of groundwater sources/sinks at different distances from a river on the groundwater discharge to that river. The sources/sinks can be changes in recharge from either instantaneous unit pulses or from unit step changes. The combined effect of individual points or areas of recharge source/sink at different locations can be added together at different times. If a groundwater salinity is assumed for each flux, then the salt load to the river can also be estimated.

The effect of recharge on baseflow to the river is initially considered for: (1) recharge at a point; and (2) recharge over a discrete rectangular area (uniformly distributed). For these cases, the solution is the complementary error function (ERFC). In addition to these 2 cases, solutions which consider the presence of a no-flow boundary, and of a sloping aquifer base are also presented. Full derivations of these solutions are presented in the Appendix.

2.1 Assumptions

Aquifer transmissivity is considered to be uniform throughout. It is assumed that there are only small fluctuations about a mean groundwater level, and that the aquifer has a horizontal base (with the exception of the sloping case). It is also assumed that the only bound on the aquifer is the river at y=0 (with the exception of the specific bounded case). Another important assumption is that any changes are small relative to the overall aquifer storage. Because of this the individual changes may be summed/combined to calculate the overall temporal stream response to more spatially distributed recharge. Sources/sinks of water at particular place and times are considered as input, with the only output being the discharge of water into/from the river. The time-lag between infiltration and recharge due to the depth of the unsaturated zone has not been considered.

2.2 Step Recharge at a Point

Recharge is considered to start at time zero and continue at a constant rate, at a point some distance (x=a) from the river (at x=0) (Fig. 1). The total flux of groundwater to the river as a result of a unit recharge change at a point x=a can be given by:

$$f_3(a,t) = -erfc\left[\frac{a}{2(Dt)^{1/2}}\right] , \tag{1}$$

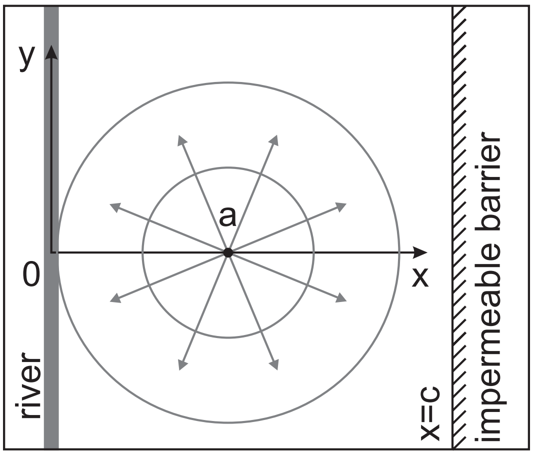

Figure 1. Effect of a recharge change at (x,y)=(a,0), expressed as a total discharge into the river along x=0. An impermeable boundary is shown at x=c (boundary is only present for Section 2.4).

where f_3 is the instantaneous flux of water to the river at time (t) as a result of a unit recharge step applied at a point with distance x=a from the river, and aquifer properties given by D = diffusivity = $K\,\bar{h}/\phi$, where K is the saturated hydraulic conductivity, $\bar{h}$ is the average height of water table, and ϕ is the porosity (Fig. 2). This point solution is the *Glover and Balmer* [1954] solution. ERFC is the complementary error function given in Section 7.1.2 of *Abramowitz and Stegun* [1965]. The Appendix contains the derivation of Eq. (1), which is given as Eq. (A12). By multiplying the value f_3 in Eq. (1) by the recharge rate due to a step point source/sink, the actual flux to the river can be calculated.

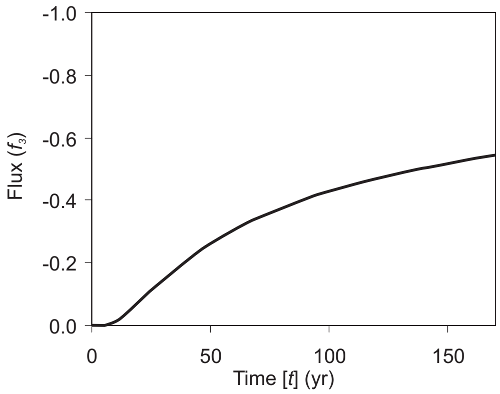

Figure 2. Cumulative groundwater flux (F_2) to the river in response to a unit pulse at x=a, t=0, given by Eq. (1) (aquifer properties were: a=8000 m and D=511,000 m^2/yr [K=5 m/d; $\bar{h}$=14 m; ϕ=0.05]).

2.3 Step Recharge Over a Rectangular Area

Recharge is considered to start at time zero and continue at a constant rate, over a rectangular area between x=a and x=b from the river (Fig. 3). The size of the rectangular area in the y direction is not important for the timing of the flux to the river, it is only important for the magnitude of the response. The groundwater flux to the river can be given by:

$$f_4(a,b,t) = -\frac{1}{(b-a)}\left\{\begin{aligned} &2\left(\frac{Dt}{\pi}\right)^{1/2}\left[exp\left(\frac{-a^2}{4Dt}\right) - exp\left(\frac{-b^2}{4Dt}\right)\right] \\ &\qquad -a\, erfc\left[\frac{a}{2(Dt)^{1/2}}\right] + b\, erfc\left[\frac{b}{2(Dt)^{1/2}}\right]\end{aligned}\right\} , \qquad (2)$$

where f_4 is the flux to the river from a unit step source of recharge distributed along a rectangular strip from x=a to x=b, with any length in the y direction at time (t) (Fig. 4). Because the distribution of the flux in the y direction is not considered, then Eq. (2) gives the response to a strip from x=a to x=b, for any length (in y direction) of recharge area (e.g. y=2 km gives twice the magnitude of response as y=1 km, although the timing of the response is the same). The Appendix contains the derivation of Eq. (2), which is given as Eq. (A14). By multiplying the value f_4 in Eq. (2) by the volumetric recharge rate due to a step source/sink (recharge rate multiplied by the area), the actual groundwater flux to the river can be calculated.

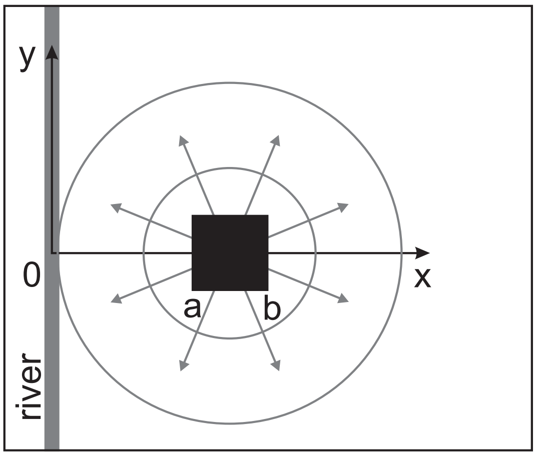

Figure 3. Effect of a recharge change over area x=a to x=b, (y can be any length) expressed as a total discharge into a river along x=0.

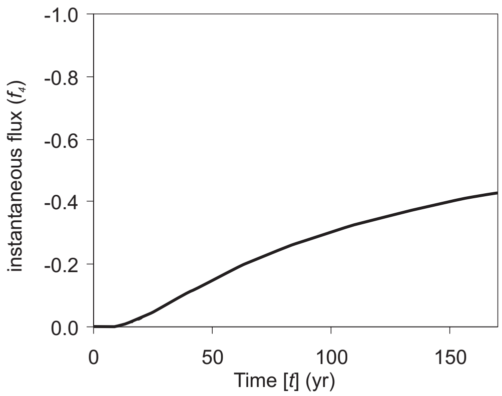

Figure 4. Flux (f_4) for a unit step over a unit length for recharge applied across a strip from x=a to x=b (aquifer properties were: a=8000 m; b=13,000 m and D=511,000 m^2/yr [K=5 m/d; $\bar{h}$=14 m; ϕ=0.05]).

2.4 Effect of a no-flow boundary

The effect of a no-flow boundary at some distance x=c (with a≤c) is now considered (see Fig. 1). This no-flow boundary will affect the groundwater flux to the river, particularly for longer times. For cases where the distance to the no-flow boundary is very large, the solution is very close to that for the unbounded region (given in Eq. 2). The solutions for the bounded region start with the corresponding solutions for the unbounded region, and then add an infinite set of positive and negative images at appropriate positions to satisfy the no-flow boundary condition at x=c and the zero height condition at x=0.

The groundwater flux to the river from a recharge pulse at a point x=a, for an aquifer with a no-flow boundary at x=c is given by:

$$f_5(a,c,t) = -\left\{ \begin{array}{l} \dfrac{a}{2t(\pi Dt)^{1/2}} exp\left(-\dfrac{a^2}{4Dt} \right) \\ + \sum_{n=1}^{\infty} (-1)^{n+1} \left[\begin{array}{l} \dfrac{2nc-a}{2t(\pi Dt)^{1/2}} exp\left(-\dfrac{(2nc-a)^2}{4Dt} \right) \\ -\dfrac{2nc+a}{2t(\pi Dt)^{1/2}} exp\left(-\dfrac{(2nc+a)^2}{4Dt} \right) \end{array} \right] \end{array} \right\} . \quad (3)$$

Fig. 5 shows the effect of different no-flow boundary positions on the flux from a unit pulse source at x=a. For the Bookpurnong case study example, the solutions to the non-bounded and bounded regions become very similar when the no-flow boundary is located at x>2.5a.

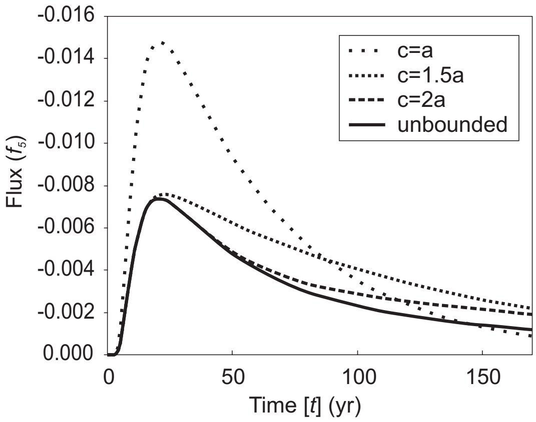

Figure 5. Effect of a no-flow boundary at different distances x=c on groundwater flux (f_5) to the river in response to a unit pulse source at t=0, as given by Eq. (3) (aquifer properties were: a=8000 m and D=511,000 m^2/yr [K=5 m/d; $\bar{h}$=14 m; ϕ=0.05]).

For a recharge step source at x=a, and a corresponding sink at x=-a, the flux at x=0 is given by integrating Eq. (3) with respect to time as:

$$f_6(a,c,t) = -\left\{ erfc\left(\frac{a}{2(Dt)^{1/2}} \right) + \sum_{n=1}^{\infty} (-1)^{n+1} \left[erfc\left(\frac{2nc-a}{2(Dt)^{1/2}} \right) - erfc\left(\frac{2nc+a}{2(Dt)^{1/2}} \right) \right] \right\} . \quad (4)$$

Fig. 6 shows the effect of different no-flow boundary positions on the flux from a unit step source at x=a. For the given example the solutions to the non-bounded and bounded regions become very similar when the no-flow boundary is located at x>2.5a. The Appendix contains the derivation of Eq. (3) and (4), which are given as Eq. (A18) and (A21).

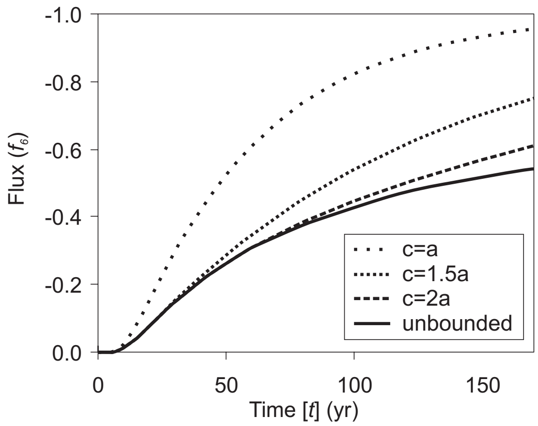

Figure 6. Effect of a no-flow boundary at different distances x=c, on groundwater flux (f_6) to the river in response to a unit step at t=0, as given by Eq. (4) (aquifer properties were: a=8000 m and D=511,000 m^2/yr [K=5 m/d; $\bar{h}$=14 m; ϕ=0.05]).

2.5 Sloping Case

The solution for a non-horizontal aquifer base is now considered. The flux to the river resulting from a recharge pulse at a point x=a, with an aquifer base angle α is given by:

$$f_7(a,t) = \frac{-a}{2t(\pi Dt)^{1/2}} exp\left[-\frac{(a-\kappa t)^2}{4Dt}\right] , \qquad (5)$$

where the diffusivity $D = K\bar{h}/\varphi$, and $K\bar{h}$ is the transmissivity (T), and where κ=K tan α / ϕ (being the physical velocity of a water particle down the slope), and α is the angle of the aquifer base. Following from this, the cumulative flux (F_7) response at x=0, to a unit pulse at x=a can be found as F_7:

$$F_7(a,t) = \int_0^t f_7(a,t)dt = -\frac{1}{2} erfc\left[\frac{a - \kappa t}{2(Dt)^{1/2}}\right] - \frac{1}{2} exp\left[\frac{a\kappa}{D}\right] erfc\left[\frac{a+\kappa t}{2(Dt)^{1/2}}\right] . \qquad (6)$$

Fig. 7 shows a range of responses for a range of aquifer base angles. The Appendix contains the derivation of Eq. (5) and (6), which are given as Eq. (A32) and (A33). For the horizontal case (α=0), Eq. (6) is equivalent to Eq. (1).

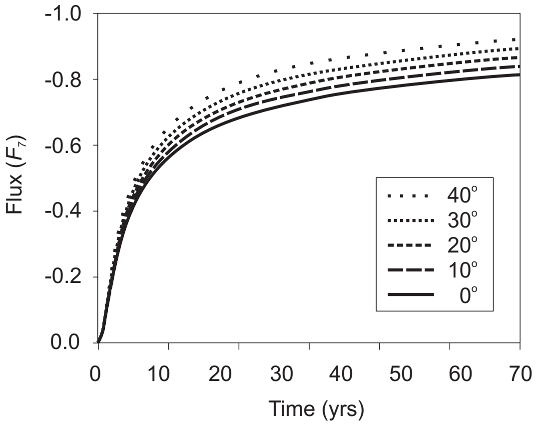

Figure 7. Effect of the slope of the aquifer base (0° is horizontal) on groundwater flux (F_7) to the river in response to a unit step at t=0, as given by Eq.(6).

3. Application to Case Study

The solution for an area (Eq. 2) has been applied to the Bookpurnong irrigation area, adjacent to the Murray River near Loxton in South Australia (SA) (Fig. 8). In this area of SA, the river forms the main drainage for groundwater and salts from the semi-arid Murray Basin (a sedimentary geologic basin approximately 300,000 km^2 in area). The Murray River begins to incise into the Basin in SA, which allows significant amounts of salt to enter the river from the Murray Basin.

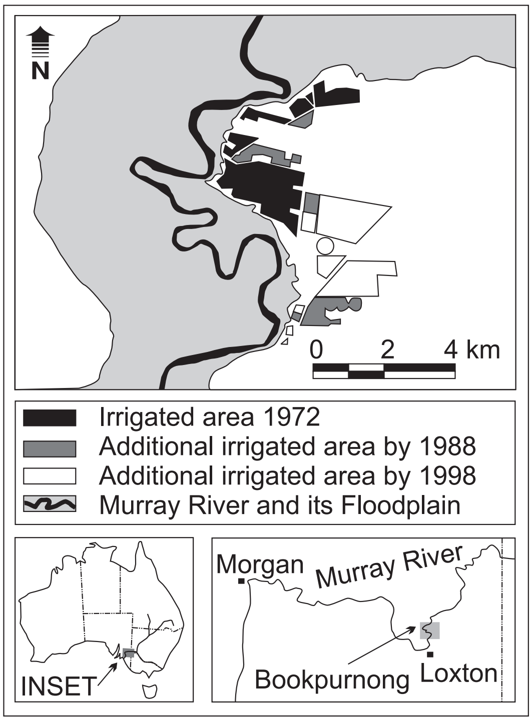

Figure 8. Location and approximate extent of the Bookpurnong Irrigation Area.

Irrigation commenced adjacent to the Murray River in South Australia in the late 1800s. The Bookpurnong Irrigation Area (between the towns of Berri and Loxton) was first developed in 1964, and has expanded since that time. Increased recharge from irrigation on areas adjacent to the river has allowed groundwater mounds to form beneath them, which has further increased the amount of saline groundwater (and hence salt load) being discharged to the floodplain and river itself. Estimating salt load accessions to the Murray River, and examining ways of managing them is critical, since the city of Adelaide relies heavily on this river for its water supply.

While there is an assumption that the river is a straight line, it has been assumed that this solution is still reasonably accurate for the Bookpurnong Irrigation Area case study. It has also been assumed that groundwater gradients are approximately perpendicular to the general direction of the floodplain, rather than to the river channel itself.

For the aquifer beneath the Bookpurnong Irrigation Area, the following representative properties were used: K=5 m/d, $\bar{h}$=14 m, ϕ=0.05 [*Watkins and Waclawik*, 1996]. Response curves were constructed from the response to recharge changes to 40 rectangular sub-areas. For each sub-area, the distances to the nearest floodplain (x=a and x=b) were determined, and its response estimated using Eq. (2), with the assumption that irrigation was uniform throughout the sub-area, and that recharge under irrigation was 200 mm/yr.

Changing irrigated area over time was determined in three stages from three aerial photo interpretations (1972, 1988, and 1998). It was assumed that: (i) 4.54 km^2 area in 1972 photo was developed between 1964 and 1972 (in 18 sub-areas); (ii) additional 1.83 km^2 area in the 1988 photo was developed between 1973 and 1988 (in 10 sub-areas); and (iii) additional 5.23 km^2 area in the 1998 photo was developed between 1989 and 1998 (in 12 sub-areas). The resulting summed water flux to the river was converted into a salt load by assuming a groundwater salinity of 20 dS/m (assumed to be 31,250 mg/L). This composite salt load curve is then shown in Fig. 9 together with recent best-estimates of salt load from the Bookpurnong area (discrete points) that were used as part of South Australia's contribution [*Australian Water Environments*, 1999] to the Murray-Darling Basin Commission's 1999 Salinity Audit [*MDBMC*, 1999].

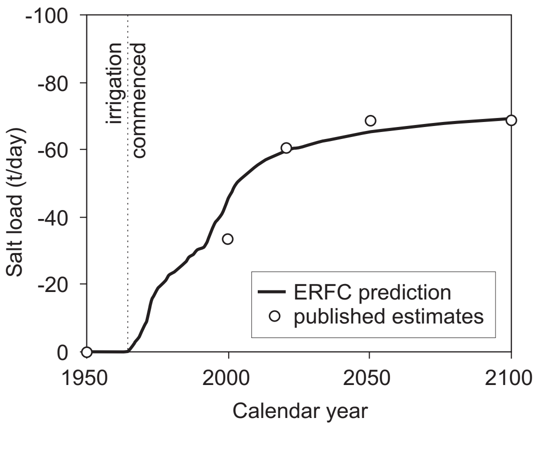

Figure 9. Estimated increase in salt load to the Murray River as a result of the Bookpurnong Irrigation Area, compared to published best-estimates [*Australian Water Environments*, 1999].

4. Discussion

The present paper provides further development of the analytical solution for stream depletion of *Glover and Balmer* [1954], and applies it to the reverse situation of discharge from an aquifer to a river as a result of recharge over a discrete area at some distance from the river (Eq. 2). While *Brutsaert* [1994] and *Verhoest and Troch* [2000a, 2000b] gave solutions for discharge from a sloping system of finite extent, they also provided special cases for discharge from a horizontal system. Their series expressions for the discharge are more complicated than the solution in the present paper, because they are for regions of finite extent (L) in the x direction. It should be noted that the present paper considers discrete areas of recharge (such as from irrigation, land-clearing), which differs from *Manga* [1999] and also the above authors, who considered spatially uniform recharge over the entire system (such as from rainfall).

The presence of a no-flow boundary at some distance greater than x=b will affect the flux to the river in the medium to long term. However, the ratio given in Eq. (A20) gives an objective measure of whether the effects of such a boundary can be ignored, or whether it will the solution will be affected during the modeled time-frame in a particular study (in which case the bounded solution in Eq. (4) can be used). A special case with a non-horizontal aquifer base has also been presented.

The simple approach used here allows the effect of groundwater discharge on river flow (and salt load) to be calculated as a result of changes in recharge, since a discharge function can be related to a particular change. Within the constraints of the assumptions, this is useful method for estimating the effects of the trading of water from one location to another, or for regional based assessment of irrigation development impacts.

5. Conclusions

The main conclusions from this paper are:

- A simple solution is presented which gives the analytical response of the groundwater flux to a river from a horizontal aquifer over time, as a result of a step change in recharge for a rectangular area along a strip x=a to b (Eq. 2). This unit approach solution can be used to sum individual responses to events at different times (both sources and sinks) to produce an overall cumulative impacts analysis.
- The importance of a particular no-flow boundary can be determined using the ratio in Eq. (A20). For cases where such a boundary has an influence on the groundwater flux within the time-frame of a study, a solution is given for both a pulse (Eq. 3) and for a step change in recharge (Eq. 4).
- The solution for a sloping aquifer base is given in Eq. (6).
- While other methods exist for such solutions, the simplicity of the solution proposed in the present paper makes it more accessible for regional assessments. The form of the solution readily allows it to be applied within spreadsheet based analyses.

Appendix A: Theoretical Derivation

The Boussinesq equation is:

$$\phi \frac{\partial H}{\partial t} = K \frac{\partial}{\partial x}\left(H \frac{\partial H}{\partial x} \right) + K \frac{\partial}{\partial y}\left(H \frac{\partial H}{\partial y} \right) + N(x,y,t) \,, \tag{A1}$$

where ϕ is the porosity, K is the hydraulic conductivity, H is the height of the water table in an unconfined aquifer, x and y are the Cartesian coordinates in the horizontal plane, N is the recharge to the aquifer, and t is time [*Bear*, 1972]. If fluctuations in the water table are small compared to the water table depth (H) then the Boussinesq equation can be linearised to give:

$$\frac{\partial h}{\partial t} = \frac{K\bar{h}}{\phi}\frac{\partial^2 h}{\partial x^2} + \frac{K\bar{h}}{\phi}\frac{\partial^2 h}{\partial y^2} + \frac{N(x,y,t)}{\phi} \,, \tag{A2}$$

where $\bar{h}$ is an average height of water table, and h = H - $\bar{h}$. Eq. 2 has the form of a diffusion equation where $K\bar{h}/\phi$ is the diffusivity (D).

In the horizontal plane, x is the distance from the stream, and y is in the direction parallel to the stream (see Fig. 1 in main text). It is assumed that there is a fixed water level at x=0, corresponding to the elevation of the stream water surface. This assumption is reasonable for a regional groundwater systems in an arid area. Initially it is also assumed that the aquifer is unbounded in the positive x direction.

General geometric configurations of sources and sinks can be considered, but firstly a unit sink/source at a point is described. This can then be summed and integrated to find the overall effect from more complex configurations if required. In the time domain, a pulse is considered to be an instantaneous source of water starting and finishing at t=0, while a step is a steady-rate source starting at t=0. The source/sink to the groundwater system causes a flux across into/from the stream at x=0. Changes in this flux are considered for a combination of unit source/sink configurations.

Unit Pulse at a Point

If only considering the whole flux into the river along the line x=0, and not the distribution of this flux in the y direction, then the problem can be treated as one dimensional.

$$\frac{\partial h}{\partial t} = \frac{K\bar{h}}{\phi}\frac{\partial^2 h}{\partial x^2} + \frac{N(x,t)}{\phi} \tag{A3}$$

The solution of Eq. (A3) corresponding to an initial value of h=0, and a unit pulse at t=0, at a point x=a, in an unbounded aquifer is:

$$h_1(x,a,t) = \frac{1}{2\phi(\pi Dt)^{1/2}} exp\left[\frac{-(x-a)^2}{4Dt}\right] \quad . \tag{A4}$$

The boundary condition for a river at x=0 is assumed to be h=0, at x=0 for all time. To satisfy this boundary condition a corresponding sink solution must be added at x=-a which is:

$$h_2(x,a,t) = -h_1(x,-a,t) = \frac{-1}{2\phi(\pi Dt)^{1/2}} exp\left[\frac{-(x+a)^2}{4Dt}\right] \quad . \tag{A5}$$

The solution which satisfies this boundary condition is $h_3=h_1+h_2$, since

$$h_3(0,a,t) = h_1(0,a,t) - h_1(0,-a,t) = 0 \quad . \tag{A6}$$

To find the flux at x=0 (the stream), the quantity:

$$-T\,\frac{\partial h_3}{\partial x} \tag{A7}$$

is evaluated at x=0. This is:

$$f_2(a,t) = \frac{-a}{2t(\pi Dt)^{1/2}} exp\left[\frac{-a^2}{4Dt}\right] \quad , \tag{A8}$$

where f_2 is the flux at the river from a pulse source at a distance a from the river. Following from this, the cumulative flux (F) response at x=0, to a unit pulse at x=a can be found as F_2:

$$F_2(a,t) = \int_0^t f_2(a,t)dt = -erfc\left[\frac{a}{2(Dt)^{1/2}}\right] \quad . \tag{A9}$$

where F_2 is the cumulative flux to the river from a unit pulse at a point.

Now, Eq. (A9) shows that the cumulative flux is a function of the dimensionless quantity (τ):

$$\tau = \frac{Dt}{a^2} \quad , \tag{A10}$$

when τ =1.0, then erfc(τ/2)=0.48, and about half of the flux has crossed the line x=0.

So the quantity τ is effectively the hydraulic response time as defined by *Gelhar and Wilson* [1974].

At large time,

$$F_2(a,t) \approx -1 + \frac{a}{(\pi Dt)^{1/2}} \quad , \tag{A11}$$

which shows that the final value of –1 is approached only very slowly. Since the region is unbounded in the positive x direction, initially half of the flux goes in the positive direction, and only returns very slowly towards the stream at x=0.

Unit Step Applied to a Point

Now a continuous step source starting at t=0 at x=a, and a corresponding step sink at x=-a are considered. The effect of this is calculated by integrating the pulse source response with respect to time to obtain the flux to the river (f_3).

$$f_3(a,t) = \int_0^t f_2(a,t)dt = -erfc\left[\frac{a}{2(Dt)^{1/2}}\right] \tag{A12}$$

Note that the cumulative flux from a unit pulse (Eq. A9) is the same as the instantaneous flux from a unit step (Eq. A12).

Unit Step Applied to a Rectangular Area

If only the one dimensional effects are looked at (i.e. the overall flux to the river and not its distribution) then it can be considered that this is also the solution for a line of sources at x=a (parallel to the stream) of unit strength per unit length. As a result, the solution for a finite area need only consider the distribution of sources in the x direction. The sources can be distributed over a finite width (x=a to x=b). Over this finite width a step source has been applied, starting at t=0, to give:

$$f_4(a,b,t) = \frac{1}{(b-a)}\int_a^b f_3(u,t)du = -\frac{1}{(b-a)}\int_a^b erfc\left[\frac{u}{2(Dt)^{1/2}}\right]du \quad , \qquad \text{(A13)}$$

$$f_4(a,b,t) = -\frac{1}{(b-a)}\left\{\begin{aligned} 2\left(\frac{Dt}{\pi}\right)^{1/2}\left[exp\left(\frac{-a^2}{4Dt}\right) - exp\left(\frac{-b^2}{4Dt}\right)\right] \\ -a\ erfc\left[\frac{a}{2(Dt)^{1/2}}\right] + b\ erfc\left[\frac{b}{2(Dt)^{1/2}}\right]\end{aligned}\right\} \quad , \qquad \text{(A14)}$$

where f_4 is the flux to the river from a step source of recharge distributed along a strip in the x direction. Because the distribution of the flux in the y direction is not considered, then Eq. (A14) gives the response to a strip from x=a to x=b, for any length (in y direction) of recharge area (e.g. y=2 km gives twice the magnitude of response as y=1 km, although the timing of the response remains the same).

Effect of no-flow boundary

We first consider a general source (S) at x=a, which can be a pulse or a step, or can also be a general function of time. A no-flow boundary at x=c is then considered, together with an image boundary at x=-c. A solution having the same source at x=a, and by symmetry having a zero flux at x=c and x=-c is:

$$\sum_{n=-\infty}^{\infty} S(4nc + a - x) + \sum_{n=-\infty}^{\infty} S((4n+2)c - a - x) \quad . \qquad \text{(A15)}$$

To meet the boundary condition at x=0, a corresponding sink solution at x=-a, and by symmetry having a zero flux at x=c and x=-c is:

$$-\sum_{n=-\infty}^{\infty} S(4nc - a - x) - \sum_{n=-\infty}^{\infty} S((4n+2)c + a - x) \quad . \qquad \text{(A16)}$$

Now since a zero height at x=0 (river) is required, then Eq. (A15) and Eq. (A16) are summed to get a solution which satisfies all boundary conditions. To get the fluxes, the sources are then summed and their derivatives taken.

So, for a pulse source at x=a (together with all the image sources and sinks), h_1 (from Eq. A4) is substituted for S in Eq. (A15 and A16). The same procedure is then followed that was carried out to obtain the flux (Eq. A8) at x=0 using the definition in Eq. (A7).

$$f_5(a,c,t) = -\sum_{n=-\infty}^{\infty} \frac{(a+4nc)}{2t(\pi Dt)^{1/2}} exp\left[-\frac{(a+4nc)^2}{4Dt}\right] - \sum_{n=-\infty}^{\infty} \frac{-a+(4n+2)c}{2t(\pi Dt)^{1/2}} exp\left[-\frac{(-a+(4n+2)c)^2}{4Dt}\right] \tag{A17}$$

By extracting the n=0 term from Eq. (A17), separating the remainder into sums of negative and positive n, and then recombining them, Eq. (A18) is obtained. These equations are mathematically equivalent, although Eq. (A18) brings out the physical meaning of the terms more clearly than Eq. (A17) does.

$$f_5(a,c,t) = -\left\{ \begin{array}{l} \dfrac{a}{2t(\pi Dt)^{1/2}} exp\left(-\dfrac{a^2}{4Dt}\right) \\ + \sum_{n=1}^{\infty} (-1)^{n+1} \left[\begin{array}{l} \dfrac{2nc-a}{2t(\pi Dt)^{1/2}} exp\left(-\dfrac{(2nc-a)^2}{4Dt}\right) \\ -\dfrac{2nc+a}{2t(\pi Dt)^{1/2}} exp\left(-\dfrac{(2nc+a)^2}{4Dt}\right) \end{array} \right] \end{array} \right\} \tag{A18}$$

The first term in Eq. (A18) is the solution for the unbounded case, and the rest of the terms correspond to an infinite set of pairs of sources and sinks (at $x = 2nc \pm a$, for n = 1, 2,…). Following from Eq. (A10), we have a dimensionless quantity (τ) from the second term in Eq. (A18):

$$\tau = \frac{Dt}{(2c-a)^2} = t/t_2 \quad , \quad t_2 = \frac{(2c-a)^2}{D} \tag{A19}$$

with t_2 the hydraulic response time which determines when time the second term (and hence the boundary) in Eq. (A18) becomes important for the solution. The ratio of t_2 to t_1 is:

$$\frac{t_2}{t_1} = \frac{(2c-a)^2}{a^2} = \left(\frac{2c}{a} - 1\right)^2 \quad . \tag{A20}$$

This provides a way to assess the relative timescale for which the presence of a particular boundary at x=c becomes important for the accuracy of the solution. The greater the value c/a, the longer it takes before the correction terms become important relative to the unbounded solution. This gives a criterion for deciding between whether to use the unbounded or bounded solution.

For a step source at x=a, and a corresponding sink at x=-a, the flux at x=0 is given by integrating Eq. (A18) with respect to time as:

$$f_6(a,c,t) = -\left\{ erfc\left(\frac{a}{2(Dt)^{1/2}}\right) + \sum_{n=1}^{\infty}(-1)^{n+1}\left[erfc\left(\frac{2nc-a}{2(Dt)^{1/2}}\right) - erfc\left(\frac{2nc+a}{2(Dt)^{1/2}}\right)\right]\right\} \tag{A21}$$

Sloping Case

The *Glover and Balmer* [1954] solution has extended to allow for the effect of a non-horizontal aquifer. As such, for a sloping case, the Boussinesq equation is:

$$\phi\frac{\partial H}{\partial t} = K\frac{\partial}{\partial x}\left(H\left(\frac{\partial H}{\partial x} + \tan\alpha\right)\right) + K\frac{\partial}{\partial y}\left(H\frac{\partial H}{\partial y}\right) + N(x,y,t) \ , \tag{A22}$$

where ϕ is the storage coefficient, K is the hydraulic conductivity, H is the height of the water table in an unconfined aquifer, x and y are the Cartesian coordinates in the horizontal plane, N is the recharge to the aquifer, α is the angle from the horizontal to the impermeable base, and t is time [*Wooding and Chapman*, 1966; *Bear*, 1972]. If fluctuations in the water table are small compared to the water table depth (H) then Eq. (A22) can be linearised to give:

$$\frac{\partial h}{\partial t} = \frac{K\bar{h}}{\phi}\frac{\partial^2 h}{\partial x^2} + \frac{K\bar{h}}{\phi}\frac{\partial^2 h}{\partial y^2} + \frac{K\tan\alpha}{\phi}\frac{\partial h}{\partial x} + \frac{N(x,y,t)}{\phi} \ , \tag{A23}$$

where $\bar{h}$ is an average height of water table, and h = H - $\bar{h}$. Eq. (A23) has the form of a diffusion equation where the diffusivity $D = K\bar{h}/\phi$, and $K\bar{h}$ is the transmissivity (T). And where κ=K tan α / ϕ is the physical velocity of a water particle down the slope.

$$\frac{\partial h}{\partial t} = D\frac{\partial^2 h}{\partial x^2} + \kappa\frac{\partial h}{\partial x} + \frac{N(x,t)}{\phi} \qquad \text{(A24)}$$

If h(x, t) is a solution of $\frac{\partial h}{\partial t} = D\frac{\partial^2 h}{\partial x^2}$ on the interval $-\infty < x < \infty$, then h(x+κt, t) is a solution of:

$$\frac{\partial h}{\partial t} = D\frac{\partial^2 h}{\partial x^2} + \kappa\frac{\partial h}{\partial x} \quad , \qquad \text{(A25)}$$

$$h(x,a,t) = \frac{1}{2\phi(\pi Dt)^{1/2}} exp\left[-\frac{(x-a+\kappa t)^2}{4Dt}\right] \quad . \qquad \text{(A26)}$$

This is a solution of Eq (A25). for a unit pulse at time zero in an unbounded sloping aquifer.

At x=0 this has the value:

$$h(0,a,t) = \frac{1}{2\phi(\pi Dt)^{1/2}} exp\left[-\frac{(-a+\kappa t)^2}{4Dt}\right] \quad . \qquad \text{(A27)}$$

A pulse at the position x=-a has the value at x=0 of:

$$h(0,\text{-}a,t) = \frac{1}{2\phi(\pi Dt)^{1/2}} exp\left[-\frac{(a+\kappa t)^2}{4Dt}\right] \quad . \qquad \text{(A28)}$$

So at x=0 the effects of these two pulses (Eq. (A27) and Eq. (A28)) are in a ratio which is fixed and independent of time:

$$exp\left[-\frac{a\kappa}{D}\right] \quad . \qquad \text{(A29)}$$

So we can construct a unit pulse solution satisfying the boundary condition h(0,a,t)=0 as:

$$h(x,a,t) = \frac{1}{2\phi(\pi Dt)^{1/2}} exp\left[-\frac{(x-a+\kappa t)^2}{4Dt}\right] - \frac{1}{2\phi(\pi Dt)^{1/2}} exp\left[\frac{a\kappa}{D}\right] exp\left[-\frac{(x+a+\kappa t)^2}{4Dt}\right] \quad . \qquad \text{(A30)}$$

To find the flux at x=0 (the stream), the quantity:

$$-T\frac{\partial h_3}{\partial x} \quad , \qquad \text{(A31)}$$

is evaluated at x=0 (where T is the transmissivity). This is:

$$f_7(a,t) = \frac{-a}{2t(\pi Dt)^{1/2}} exp\left[-\frac{(a-\kappa\, t)^2}{4Dt}\right] \quad , \qquad \text{(A32)}$$

Following from this, the cumulative flux (F_7) response at x=0, to a unit pulse at x=a can be found as F_7:

$$F_7(a,t) = \int_0^t f_7(a,t)dt = -\frac{1}{2} erfc\left[\frac{a-\kappa\, t}{2(Dt)^{1/2}}\right] - \frac{1}{2} exp\left[\frac{a\kappa}{D}\right] erfc\left[\frac{a+\kappa\, t}{2(Dt)^{1/2}}\right] \quad . \qquad \text{(A33)}$$

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