Hydrogeology and Aquifer Tidal Propagation in Cockburn Sound, Western Australia

A.J. Smith and W.P. Hick

CSIRO Land and Water
HYDROGEOLOGY AND AQUIFER TIDAL PROPAGATION IN COCKBURN SOUND, WESTERN AUSTRALIA

by

A. J. Smith and W. P. Hick

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ABSTRACT

Harmonic analysis of aquifer tidal propagation in Cockburn Sound, Western Australia, is applied with a tidal method (Ferris, 1951) to obtain estimates of the aquifer diffusivity. The work was carried out as part of the SCOR/LOICZ submarine groundwater discharge (SGD) Intercomparison Experiment, which was hosted by CSIRO Land and Water during the period 24 November to 8 December 2000. The dominant diurnal tidal frequencies (K1 and O1 constituents) were found to propagate more than 150 m into the coastal aquifer as a result of the high transmissivity of the coastal sands and very high transmissivity of the limestone that underlies them. Based on an analysis of tidal efficiency and lag, the bulk hydraulic conductivity values for the sand unit vary in the range 53 to 174 m.day\(^{-1}\). These values are characteristic of medium to coarse sand. The applied technique was not suited for estimating diffusivity values for the limestone because the assumptions of the tidal method are inconsistent with the field conditions.
MATHEMATICAL NOTATION

Variables

\( F \)  Tidal forcing function (general)
\( H \)  Head at a prescribed tidal boundary [L]
\( P \)  Period [T]
\( R \)  Tidal response function (general)
\( S \)  Aquifer storage coefficient [1]
\( S_o \)  Aquifer specific storativity [L\(^{-1}\)]
\( T \)  Aquifer transmissivity [L\(^{2}\)T\(^{-1}\)]
\( t \)  Time [T]
\( x \)  Horizontal spatial coordinate [L]
\( y \)  Time series (general)
\( \delta t \)  Time-series sampling interval [T]
\( \Phi \)  Fourier coefficient (complex)
\( \omega \)  Angular frequency [T\(^{-1}\)]
\( \nu \)  Frequency [T\(^{-1}\)]

Subscripts

\( a \)  Amplitude
\( i \)  Discrete value
\( j \)  Fourier harmonic
\( k \)  Tidal constituent
\( N \)  Nyquist harmonic
\( \theta \)  Phase
INTRODUCTION

During the two-week period from November 24 to December 8, CSIRO Land and Water (CLW) hosted an intercomparison experiment to assess different techniques for estimating Submarine Groundwater Discharge (SGD) to the near-shore marine environment. The experiment was conducted in Cockburn Sound, Western Australia, and is the first of five SGD intercomparisons being planned and coordinated by the SCOR/LOICZ Working Group 112. The Scientific Committee on Oceanographic Research (SCOR) and the Land-Ocean Interaction in the Coastal Zone (LOICZ) Project are both coordinated through the International Council of Scientific Unions.

The field site (Figure 1) selected for the first SGD intercomparison was located within Northern Harbour at the northern end Cockburn Sound, Western Australia. This locality is approximately 10 kilometres south of Fremantle Port.

Transects of measurement stations extending onshore and offshore from the beach in Northern Harbour were established and multiple methods were employed to estimate SGD. The main instruments transect contained seven groundwater-monitoring bores (Figure 1) in which water levels were continuously logged at ten-minute intervals.

This report presents the calculation of tidal efficiencies and lags for the intercomparison monitoring bores and describes the application of a tidal method (Ferris, 1951) to estimate the diffusivity of the shallow coastal aquifer at the study site. The work was initiated after it was found that prominent tidal water level responses could be detected in the aquifer more than 150 metres from the shore. This indicated very high aquifer transmissivity, semi-confinements of groundwater or possibly both, and provided an opportunity to obtain estimates of the aquifer hydraulic properties.

A review of the hydrogeology was carried out as a pre-requisite to this work and a brief overview is presented in this report.
Figure 1: Site Location Map
HYDROGEOLOGY

Regional Setting

The Perth Basin lies to the west of the Darling Fault (Figure 2) and contains up to 12,000 meters of marine and continental sediments. The stratigraphic sequence to a depth of around 2,000 metres below the present land surface elevation contains Jurassic (210-140 Ma) and Cretaceous (140-65 Ma) age sediments that are overlain by a relatively thin veneer of late Tertiary to Quaternary age sediment. The Tertiary and Quaternary deposits are known locally as the superficial formations and, collectively, vary in thickness up to a maximum of approximately 110 meters.

The saturated extent of the superficial formations forms a mostly unconfined aquifer system that is called the Superficial Aquifer. Recharge to the aquifer occurs through direct vertical infiltration of rainfall, whereas groundwater drains primarily by lateral flow through the aquifer. There is some leakage of groundwater to the confined aquifer system beneath but most groundwater discharges eventually to the Indian Ocean, Swan-Canning River estuary system and other regional surface drains. The result is a number of distinct regional groundwater mounds and flow systems that are located between the regional discharge boundaries (Figure 3).

The Cockburn Sound study area lies mostly on the western edge of the Jandakot Mound and partly within the Safety Bay Mound. Over most of the study area, the superficial formations have a saturated thickness of around 25 to 30 metres and unconformably overlie an erosional surface of Cretaceous sediments that act as a low-permeability base to the Superficial Aquifer (Figure 4). In the southern part of the study area, the superficial formations are underlain by Rockingham Sand, which occupies an erosional channel in the Cretaceous sediments. This is known to be more than 100 metres deep.

Exchange of groundwater between the Superficial Aquifer and Cretaceous-Jurassic artesian aquifer system is regionally variable and leakage between these two systems is only crudely understood at the local scale.

Subcrop Formations

The superficial formations mostly overlie Cretaceous sediments, which include the Kardinya Shale Member (Osborne Formation) and Pinjar and Waneroo Members (Leederville
Formation). A different situation exists in the southern part of Cockburn Sound where the Rockingham Sand underlies the superficial formations. The interpreted regional extent of subcrop units (Davidson 1995) is based on drilling data from the mainland and offshore on Garden Island; see Figure 4 - Sections A, B and C. The following hydrogeological descriptions of the subcrop units are based on the synthesis of Davidson (1995).

**Kardinya Shale Member**

The Kardinya Shale Member consists of moderately to tightly consolidated, interbedded siltstones and shales, which are typically dark green to black, and includes thin interbeds of mostly fine-grained sandstone. It has relatively low permeability and is considered to be an aquitard the separates the Superficial Aquifer from the deeper Leederville Aquifer. Leakage through the Kardinya Shale Member is considered to be negligible compared to aquifer flows.

**Pinjar and Waneroo Members**

The Pinjar and Waneroo Members are upper units within the Leederville Aquifer and consist of discontinuous, interbedded sandstones, siltstones and shale of marine and non-marine origin. The subcrop of these two units beneath the superficial formations represents an area of direct contact between the Superficial and Leederville Aquifers. The intercomparison study area is a region of upward hydraulic gradient where there is potential for the Leederville Aquifer to discharge to the Superficial Aquifer.

**Rockingham Sand**

The Rockingham Sand occupies what is thought to be an erosional channel incised into the Cretaceous sediments, which extends from Rockingham to offshore beneath the southern end of Garden Island. The unit consists of slightly silty, medium-grained to coarse-grained sand of shallow marine origin. The maximum thickness of Rockingham Sand is approximately 110 metres at the southern end of Cockburn Sound in the Rockingham area.
**Superficial Formations**

Individual stratigraphic units that comprise the superficial formations in the Cockburn Sound area are—in order of deposition—the Tamala Limestone, Cooloongup Sand, Becher Sand and Safety Bay Sand (Figure 5 - Sections A, B and C). These are known collectively as the Kwinana Group.

**Tamala Limestone**

The Tamala Limestone unit is a calcareous eolianite that unconformably overlies Cretaceous sediments and Rockingham Sand. It contains various proportions of quartz sand, fine-grained to medium-grained shell fragments and minor clay lenses. The limestone typically exhibits secondary porosity in the form of numerous solution channels and cavities. The average base elevation of Tamala Limestone in the intercomparison study area is around -25 to -30 metres Australian Height Datum* (mAHD); see Figure 4.

Garden Island is an offshore outcrop of Tamala Limestone and is part of a former ‘drowned’ dune ridge called the Garden Island Ridge (Seale et al. 1988) which runs northwest through Point Peron, Garden Island, Carnac Island and Rottnest Island. A roughly parallel sand and limestone ridge (Spearwood Ridge) is located onshore from the coast along the contact between the Tamala Limestone and Safety Bay Sand. The area between these ridges is known as the Warnbro-Cockburn Depression. Tamala Limestone also outcrops as submarine reef within this depression.

Passmore (1970, Figure 37) depicts a section through the Tamala Limestone and overlying sediments from the southern end of Garden Island to the mainland across Cockburn Sound. The section is along a similar line to Section C in this report and is based on drilling logs from Fremantle Port Authority Line 1 No. 7 (FPA7), Line 10 No. A (FPA10A) and No. F1 (FPAF1). The base of the Tamala Limestone occurs at approximately -25 to -30 mAHD at these locations, which is consistent with drilling logs from the mainland.

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* Zero metres Australian Height Datum (0 mAHD) is approximately equal to mean sea level.
Cooloongup Sand

The Cooloongup Sand unit consists of fine-grained to coarse-grained feldspathic quartz sand of grey and yellow-brown colour, with variable amounts of shell material (up to 25%). It unconformably overlies the Tamala Limestone.

Becher Sand

Becher Sand originated in the near-shore marine environment and consists of grey, fine-grained to medium-grained quartz and skeletal sand that is mostly structureless and bioturbated. It unconformably overlies the Tamala Limestone and is unconformably overlain by the Safety Bay Sand. Becher Sand extends along the coastal margin and is typically 10 to 15 metres thick. The base of the unit can locally contain a layer of silty calcareous clay that is rich in shell fragments.

Safety Bay Sand

Safety Bay Sand unconformably overlies Tamala Limestone and Becher Sand and consists of cream, unlihified, calcareous fine-grained to medium-grained quartz sand and shell fragments. Traces of fine-grained, black heavy minerals are also present. Safety Bay Sand is clearly visible along the coastal margin as white, aeolian sand dunes, which extend offshore into Cockburn Sound as submarine bank units.

Groundwater Occurrence and Flow

Artesian Aquifer System

Artesian aquifers of the Perth Basin comprise a multi-layered aquifer system that is recharged by slow downward leakage of groundwater through overlying formations. This system extends offshore beneath the seabed and is thought to discharge to the ocean at least several kilometers from the coast, possibly through offshore faults. The precise nature of artesian submarine discharge, however, is unknown and beyond the scope of this report.
Rockingham Aquifer

The Rockingham Aquifer is defined in association with the Rockingham Sand and is locally confined by discontinuous clay lenses at the base of the superficial formations. Confinement of groundwater in the Rockingham Aquifer is indicated by propagation of ocean tidal fluctuations more than 150 metres inland from the shoreline (Passmore, 1970). This suggests elastic expansion and compression of the aquifer under the load of the outgoing and ingoing tide.

Because the Rockingham Aquifer is thicker than the Superficial Aquifer, the saltwedge can penetrate deeper. The bottom part of the aquifer contains seawater to an elevation of around -65 mAHD, while the top 40 metres contain groundwater of salinity less than 1,000 mg/L (Davidson, 1995). Groundwater discharges to the ocean over the saltwater interface. The Rockingham Aquifer is assumed to have similar hydraulic conductivity to the superficial formations.

Superficial Aquifer

Groundwater in the Superficial Aquifer (Jandakot and Safety Bay Mounds) flows generally in a westerly direction and discharges to the near shore marine environment along the coastline at Cockburn Sound and Safety Bay. Recharge occurs through infiltration of rainfall and there is virtually no surface drains because the surface sands are permeable enough to prevent significant surface runoff.

The coastal strip of the Superficial Aquifer is characterised by very high transmissivity—due to secondary porosity in the Tamala Limestone—and has very small horizontal hydraulic gradients. Several kilometers inland from the coast, there is a relatively narrow band of lower permeability sediments that run roughly parallel to the coastline along the contact between the Bassendean Dunes and Tamala Limestone. A dramatic change in hydraulic gradient is the main evidence for the existence of this flow barrier. The East Beeliar Wetlands, a north-south chain of lakes and wetland located approximately 5 kilometres from the coast, are a surface expression of higher groundwater levels that are ‘dammed’ behind this barrier. Water levels in these lakes are up to 18 metres above mean sea level.

Horizontal hydraulic gradients in the coastal strip are as small as 0.0002—that is, a 1 m change in water level over a 5 km horizontal distance—but are larger than 0.01 (50 times larger) across some parts of the flow barrier. Nield (1999, unpublished) attributes these
steep gradients to the presence of clay in the Tamala Limestone, as indicated in hydrogeological logs from a dense network of monitoring bores in the vicinity of Aloca’s residue disposal areas. The fact that horizontal hydraulic gradients vary by a factor of approximately 50 indicates that there is a similar order of magnitude variation in the Superficial Aquifer transmissivity and hydraulic conductivity.

Based on calibration of a groundwater recharge and flow model, Nield (1999, unpublished) estimates that the hydraulic conductivity of the Superficial Aquifer in the Cockburn Groundwater Area varies spatially in the range 6 to 1,660 m/day, for the assumption of low rainfall recharge, and 8 to 3,000 m/day for the assumption of high rainfall recharge. High value estimates of hydraulic conductivity are for the coastal strip and low value estimates are for the flow barrier.

Along the coastal margin of the superficial formations, groundwater in the Tamala Limestone is locally confined where the shelly clay layer at the base of the Becher Sand is present. This layer, commonly referred to as ‘basal clay’ or ‘shell bed’ in drilling logs, is in the order of 1 metre thick. Vertical hydraulic head differences between groundwaters in the Tamala Limestone and Safety Bay Sand, and propagation of tidal fluctuations hundreds of metres into the Tamala Limestone are the main evidence that this unit is confined in some locations. The Safety Bay Sand and Becher Sand units contain unconfined groundwater, therefore tidal fluctuations are rapidly attenuated inshore from the coast. Davis et al. (1994) observed diurnal tidal fluctuations in piezometric head of up to 0.4 metres in the Tamala Limestone, whereas tidal groundwater level fluctuations in the overlying sands were negligible at the same locations.

**Saltwater Intrusion**

Saltwater is known to intrude up to 2 kilometres inland from the coast in the superficial formations due to very high transmissivity in the Tamala Limestone. Because of this effect, the saltwater interface is almost flat and the saltwater wedge is observed as a thin layer of saline water at the base of the Superficial Aquifer within the Tamala Limestone. Where the Tamala Limestone is locally confined by clay at the base of the Becher Sand a much smaller saltwedge is also present in the sand units above the clay. The saltwedge configuration at the northern end of Cockburn Sound—based on measured groundwater salinities in monitoring bores—is indicated in Section D (Figures 2 to 5). There is also saltwater intrusion into the Rockingham Aquifer at the southern end of Cockburn Sound.
Intercomparison Site (Northern Harbour)

The site selected for conducting the SCOR/LOICZ Intercomparison experiment is located in Northern Harbour at the northern end of Cockburn Sound (Figure 1). The coastal hydrogeology at this location is depicted at a regional scale in Figures 2 to 5 - Section A and at smaller scale in Figure 6 - Section D.

The base of the superficial formations in this area is at an elevation of around -30 mAHD and the saturated thickness of the Superficial Aquifer is approximately 30 metres. Regional hydraulic gradients in the inland direction are around 0.0003, which is equivalent to a 1-metre rise in the groundwater table over a 3.5 kilometre distance.

Calibrated hydraulic conductivity values from groundwater flow modelling by Nield (1999, unpublished) suggest that this region of the Cockburn Groundwater Area has the highest transmissivity. In the modelling, the Tamala Limestone and overlying sand units are combined as a single aquifer layer. Vertically averaged hydraulic conductivity values of between 1,600 and 3,000 m/day were used to simulate the very small hydraulic gradients that are observed in the field. The particular values used in a simulation depended upon the assumed rates of groundwater recharge. The above range of values corresponds to the range of aquifer recharge rates that were considered to be realistic in comparison with rainfall. The large distance inland that the ocean saltwedge penetrates is further evidence of very high transmissivity in the Tamala Limestone.

Because the Superficial Aquifer is around 30 metres thick were it discharges to the ocean, it is expected that most of the freshwater discharge occurs close to the shoreline, probably within 30 metres of the beach.

Drilling

Two new piezometer nests (NH1 and NH2) were installed at Northern Harbour to monitor inshore groundwater levels during the intercomparison experiment; their locations are indicated in Figures 1 and 6. At each location there is one deep piezometer, approximately 17 metres deep and slotted over the bottom 6 metres, and one shallow piezometer, approximately 5 metres deep and slotted over the bottom 3 metres. All piezometers were constructed using 50 mm diameter PVC pipe. Bore completion details are presented in Table 1.
All installations were carried out by CSIRO Land and Water on 15th November 2000 using an auger drill and ‘solid’ auger. Because the water table was close to the surface and most drilling was through fully saturated and unconsolidated sediment, this method did not allow accurate logging of the stratigraphy. It was possible to determine the depth to limestone based on drilling resistance but the sand sequence above the limestone can only be described generally. Some interpretation of stratigraphy was possible during retrieval of the auger sections by examination of sediments adhered to the auger blades.

The first attempt to install the deep piezometer at location NH2 failed during drilling, when the auger plug became stuck and the top of the hole ‘slumped’ due to the drilling spoil presumably entering cavities in the limestone. A new deep hole (NH2a) was drilled within 2 metres of the abandoned hole.

Limestone was encountered at approximately 14 metres below ground level (m BGL) in the abandoned hole at NH2 and at approximately 19 m BGL in NH2a. The depth to limestone at NH1a is approximately 18 m BGL.

In general, the sand sequence above the limestone became coarser and muddier with increasing depth, varying from Safety Bay Sand at the surface to coarse and very coarse, dark grey muddy sand above the limestone. The muddy sand contained rounded grain up to 2 mm diameter, shell fragments and sparse plant fibre. The bottom of NH2 contained yellow, coarse and very coarse muddy sand, with rounded grains up to 3mm diameter.

The interpreted stratigraphic sequence at the site is Tamala Limestone, ?Cooloongup Sand, Becher Sand and Safety Bay Sand.
Table 1: Bore completion details for sites NH1 and NH2

<table>
<thead>
<tr>
<th>Bore ID</th>
<th>Easting (Degrees, decimal minutes)</th>
<th>Northing (Degrees, decimal minutes)</th>
<th>Depth (m BGL)</th>
<th>Screen Length (m)</th>
<th>Ground Surface RL (m AHD)</th>
<th>Top of Pipe RL (m AHD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH1a (Deep)</td>
<td>32 08.356</td>
<td>115 45.863</td>
<td>16.44</td>
<td>6.0</td>
<td>1.414</td>
<td>1.721</td>
</tr>
<tr>
<td>NH1b (Shallow)</td>
<td>32 08.358</td>
<td>115 45.863</td>
<td>5.44</td>
<td>3.0</td>
<td>1.444</td>
<td>1.944</td>
</tr>
<tr>
<td>NH2a (Deep)</td>
<td>32 08.399</td>
<td>115 45.840</td>
<td>18.76</td>
<td>6.0</td>
<td>1.046</td>
<td>1.454</td>
</tr>
<tr>
<td>NH2b (Shallow)</td>
<td>32 08.397</td>
<td>115 45.840</td>
<td>5.30</td>
<td>3.0</td>
<td>1.009</td>
<td>1.412</td>
</tr>
</tbody>
</table>
Figure 2: Regional Geomorphology
Figure 3: Superficial Aquifer Groundwater Areas
Figure 4: Subcrop Formations to the Superficial Aquifer
Figure 5: Coastal Geology

Legend:
- Environmental Geology
- Sand (Limestone)
- Sand (Alluvium)
- Limestone (Safety Bay Sand)
- Safety Bay Sand
- Terrigal Limestone

Data Sources:
- 1:50,000 Environmental Geology Series (DOE)

1:25,000

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SCOR/LOICZ SGD Intercomparison
Figure 6: Regional Geomorphology
Section A: Hydrological cross section through the northern end of Garden Island and Cockburn Sound; see Figures 2 to 5
Section B: Hydrological cross section through the southern end of Garden Island and Cockburn Sound; see Figures 2 to 5
Section C: Hydrological cross section through the southern end of Garden Island and Cockburn Sound; see Figures 2 to 5
Section D: Hydrological cross section through the SCOR/LOICZ SGD Intercomparison site; see Figure 6
THE TIDAL METHOD

The tidal method (Ferris, 1951; Carr and Van Der Kamp, 1969; Townley, 1995) is a simple technique for estimating aquifer diffusivity— the ratio of aquifer transmissivity to the aquifer storage coefficient— based upon the response of the aquifer to tidal forcing at a boundary. Similar to conventional pump test techniques, in which the piezometric head response of an aquifer to artificial pumping is used as a basis for estimating aquifer hydraulic properties, the tidal method takes advantage of natural tidal forcing and obtains an estimate of diffusivity from the attenuation of a tidal signal as it propagates into an aquifer. The tidal signal can be expressed as a linear combination of sinusoidal terms or tidal constituents (Forman and Henry, 1989, p.109) that are differentially attenuated as they travel through the aquifer; the degree and rate of attenuation depends upon the aquifer hydraulic properties. There are two possible mechanisms for tidal propagation: (1) flow of tidal water into and out of an unconfined aquifer, (2) compression and expansion of a confined aquifer due to the load of the incoming and outgoing tides. In an aquifer that has tidal forcing at a lateral boundary, the attenuation of the tidal signal is normally described by the two quantities tidal efficiency and lag.

Tidal efficiency, $TE_k$, is a normalised measure that relates the amplitude of head fluctuations in the aquifer to the amplitude of fluctuations at the tidal boundary. In general,

$$TE_k = \frac{R_{ak}}{F_{ak}}$$  

where $F_a$ is the amplitude of forcing at the tidal boundary, $R_a$ is the amplitude of the response at a point in the aquifer and $k$ denotes the tidal constituent or frequency. The tidal efficiency has a value between zero and one.

Lag is an inverse measure of the velocity of propagation of a tidal constituent as it moves through the aquifer

$$lag_k = R_{\theta k} - F_{\theta k}$$  

where $\theta_F$ is the phase of tidal forcing and $\theta_R$ is the phase of the tidal response at a point in the aquifer. The slower the speed of propagation is, the larger the lag.
Ferris (1951) presents analytical expressions that describe the tidal efficiency and lag in a one-dimensional, semi-infinite and homogeneous aquifer with uniform transmissivity (see also Carr and Van Der Kamp, 1969). Inverse solutions are obtained for 1D-transient groundwater flow with a tidal or harmonic boundary condition at \( x = 0 \)

\[
\frac{\partial^2 h}{\partial x^2} = \frac{S}{T} \frac{\partial h}{\partial t}
\]

(3)

\[
h(0,t) = H_a \cos(\alpha x - H_o)
\]

(4)

Here, \( H_a \) is the amplitude of head fluctuations at the tidal boundary \([\text{L}]\) and \( H_o \) is the phase measured in radians. Since (3) and (4) are both linear, the groundwater flow problem they describe can be decomposed into a steady-state flow problem and a harmonic flow problem that consists of one or more frequencies. The harmonic solution is rearranged to yield the following expressions for tidal efficiency and lag

\[
TE_k = \exp\left(-x \sqrt{\frac{\pi S}{TP_k}}\right)
\]

(5)

\[
\text{lag}_k = x \sqrt{\frac{SP_k}{4\pi T}}
\]

(6)

The coordinate \( x \) represents distance from the tidal boundary, \( S \) is the aquifer storage coefficient \([1]\), \( T \) is a uniform aquifer transmissivity \([\text{L}^2\text{T}^{-1}]\), \( P_k \) is the period \([\text{T}]\) of the tidal constituent \( k \) and \( \omega_k = 2\pi/P_k \) is the corresponding angular frequency \([\text{T}^{-1}]\). Since a particular tidal signal may contain several dominant frequencies, e.g. semi-diurnal and diurnal constituents, several independent estimations of diffusivity are possible from a single tidal signal. In general, longer period constituents will propagate further into the aquifer and yield estimates of diffusivity that apply to larger regions of the aquifer.

From (5) and (6), efficiency-based and lag-based expressions for aquifer diffusivity are straightforward to derive

\[
\frac{T}{S} = \frac{\pi x^2}{(\ln TE_k)^2 P_k}
\]

(7)
Equating (7) and (8) gives the non-dimensional relationship

$$\frac{\text{lag}_k}{P_k} = -\left(\frac{1}{2\pi}\right) \ln \text{TE}_k$$

(9)

where the left-hand term is a normalised lag. It follows that, on a semi-log plot, the normalised lag and tidal efficiency are related by the straight line with slope equal to $-1/2\pi$. The tidal efficiency decreases exponentially with increasing distance from the tidal boundary, whereas the lag increases linearly.

**FOURIER ANALYSIS**

The term Fourier analysis refers to any data analysis technique that describes or measures fluctuations in a time series by comparing them with sinusoids. Well known methods include filtering, least squares regression on sinusoids and harmonic analysis (Bloomfield, 1976; Chatfield, 1975; James, 1995). Fourier techniques are commonly used to detect frequency components that are hidden within a 'noisy' signal.

Harmonic analyses fits uniformly sampled time series data to a set of harmonic frequencies; that is, a set of frequencies that are integer multiples of one another. The fitted harmonics are normally in the frequency range $0 \leq \nu \leq \nu_N$, where

$$\nu_N = \frac{1}{P_N} = \frac{1}{2\delta t}$$

(10)

is the Nyquist frequency [T$^{-1}$], $P_N$ is the Nyquist period [T] and $\delta t$ is the data sampling interval [T].

The Nyquist condition simply states that the smallest period that can be reasonably fitted to the data is equal to twice the sampling interval. This corresponds to fitting, at most, one local maxima or minima between each pair of data point in the time series. From a practical point of view, the Nyquist conditions tells us that it is unreasonable to fit, say, a semi-diurnal frequency (12 hour period) to data that has only been sampled 24 hourly. The smallest period and highest frequency that should be fitted to diurnal data are 48 hours and 0.5 cycles per day, respectively.
**Discrete Fast Fourier Transform (FFT)**

The discrete FFT is a computationally efficient algorithm that enables regularly sampled time series data to be fitted by a set of harmonic frequencies (Chatfield, 1975; Bloomfield, 1976; James, 1995). For a time series of \(n\) data values, the FFT fits \(n/2\) frequencies over the frequency range \(2\nu_n / n \leq \nu \leq \nu_n\). The FFT of the time series is the set of \(n/2\) complex valued Fourier coefficients \(\Phi(v_j)\), where the moduli \(|\Phi(v_j)|\) are equal to the amplitudes of the fitted frequencies and arguments the \(\arg(\Phi(v_j))\) are equal to the phases.

The Inverse Fast Fourier Transform (IFFT) is a reverse operation that converts a set of Fourier coefficients \(\Phi(v_j)\) back to the original time-series data \(y(t_i)\). This relationship is denoted symbolically as (James, 1995)

\[
y(t_i) \leftrightarrow \Phi(v_j)
\]

A measure of the energy or power at each FFT frequency is provided by the Power Spectral Density (PSD) function

\[
S(v_j) = \Phi(v_j) \Phi^*(v_j)
\]

Here, \(\Phi^*(v_j)\) denotes the complex conjugates of the Fourier coefficients. A large value of \(S(v_j)\) indicates more energy at frequency \(v_j\). This means that the dominant frequencies in a time series are represented as larger spikes on a PSD plot.

**TIDE AND WATER LEVEL DATA**

**Fremantle**

The tide station at Fremantle—maintained by the Department of Transport, Western Australian—is the nearest tidal monitoring location to Cockburn Sound. Historic tide data is available upon request to the Maritime Division at a minimum continuous sampling interval of five minutes. Fremantle tide data (Figure 7) used in this report are sampled at 10-minute intervals over the period 01/09/2000 to 16/01/2001.
Fremantle has a mixed tide (Figure 7) that is characterised by diurnal (24-hour) and semi-diurnal (12-hour) constituents. The K1† (period = 23.9345 hours) and O1‡ (period = 25.8193) diurnal constituents are the dominant frequencies (Department of Transport, unpublished harmonic analysis). The semi-diurnal frequencies vanish for part of each month, which makes them unsuited for use with the tidal method.

Cockburn Sound

Data Collection

Tide data was logged continuously at 10-minute intervals for the duration of the SGD intercomparison experiment. Water levels were measured using a capacitive water level probe (Dataflow Systems) and model 392 data recorder (Dataflow Systems) suspended within a 50-millimetre PVC stilling well and mounted off a sea wall at the intercomparison site. The same equipment was used to record water levels—also at 10-minute intervals—in each of the five monitoring bores. Logger internal clocks were synchronized using the same laptop computer so that the tide and water levels in all bores were recorded at the same ten-minute intervals.

The tide data collected during the intercomparison is presented in Figure 8. Because the tidal logger failed on 12 December and the length of tidal record obtained to that point in time was too short, the K1 and O1 tidal constituents could not be separated in a harmonic analysis. Department of Transport tide data for Fremantle (Figure 7a) was used to ‘repair’ the Cockburn Sound data set (Figure 8b) and extend the length of the time series to match that obtained for the groundwater monitoring bores.

Harmonic Analysis

All data sets presented in this report are standardised to a ten-minute sampling interval. The corresponding 72 cycles per day Nyquist frequency is more than adequate to detect the expected diurnal or 1 cycle per day tidal constituents. The number of harmonics fitted in each analysis depends upon the number of data values in the time series being considered. Since the discrete FFT fits \( n/2 \) harmonics to \( n \) data points, a larger number of data points

† K1 is the lunisolar diurnal constituent; with O1 it expresses the effect of the Moon's declination
‡ O1 is the lunar diurnal constituent
result in more frequencies and a higher resolution PSD plot. When a standardised sampling interval is used, the total length of the time series determines the number of data points and Fourier frequencies. All data sets used in this report contained 4628 data points, corresponding to the period from 25/11/2000 @ 17:10 to 27/12/2000 @ 20:20. This means that 2314 frequencies are fitted in the harmonic analyses presented.

Foreman and Henry (1989, Table 1) specify that the minimum total length of time series required for separation of the K1 and O1 tidal constituents in a harmonic analysis are 24 hours and 328 hours (13.67 days), respectively. The data sets used in this report are approximately 32 days long, which satisfies the above criteria. Harmonic analyses are performed using Mathematica 3.0 (Wolfram Research, 1988-1996) by first detrending the time series—that is, subtracting the mean data value from each data value—and then implementing the intrinsic FFT function “Fourier[list]”. The PSD function is then calculated as \( S(v_j) = \Phi(v_j) \Phi^*(v_j) / n \).

Figures 9 to 14 depict water levels and the PSD functions for six of the seven intercomparison monitoring bores; NH1a, NH2a, NH2b, NH3, NH4 and NH5. A continuous record was not obtained in NH1b due to problems with the data logger.

The ‘spectral signatures' of these data sets are consistent with the tide data and the K1 and O1 tidal constituents are clearly discernable. Peaks on the PSD plots at lower frequencies (\( v_j \leq 0.25 \)) correspond to the passage of weather systems, associated fluctuations in barometric pressure and 'piling up' of water at the shoreline due to prevailing onshore winds. These weather effects are non-stationary, however, and are complicated by the fact that barometric pressure fluctuations affect the aquifer storage directly through atmospheric loading, as well as indirectly through tidal loading. They are therefore not suited for use with the tidal method.
Figure 7: Tidal data and Power Spectral Density for Fremantle for the period 1/9/2000 to 16/1/2001; sampling interval is 10 minutes
Figure 8: Cockburn Sound tidal data and ‘repaired’ Cockburn Sound tidal data for the period 25/11/2000 to 3/1/2001; sample interval is 10 minutes
Figure 9: Hydrograph and Power Spectral Density for groundwater monitoring bore NH1a for the period 25/11/2000 to 27/12/2000; sampling interval is 10 minutes
Figure 10: Hydrograph and Power Spectral Density for groundwater monitoring bore NH2a for the period 25/11/2000 to 27/12/2000; sampling interval is 10 minutes
Figure 11: Hydrograph and Power Spectral Density for groundwater monitoring bore NH2b for the period 25/11/2000 to 27/12/2000; sampling interval is 10 minutes.
Figure 12: Hydrograph and Power Spectral Density for groundwater monitoring bore NH3 for the period 25/11/2000 to 27/12/2000; sampling interval is 10 minutes.
Figure 13: Hydrograph and Power Spectral Density for groundwater monitoring bore NH4 for the period 25/11/2000 to 27/12/2000; sampling interval is 10 minutes
Figure 14: Hydrograph and Power Spectral Density for groundwater monitoring bore NH5 for the period 25/11/2000 to 27/12/2000; sampling interval is 10 minutes
AQUIFER TIDAL PROPAGATION

Tidal Efficiency, Lag and Aquifer Diffusivity

At each monitoring bore, applying the tidal method to the K1 and O1 tidal constituents gives four estimates of aquifer diffusivity; each constituent yields an efficiency-based and lag-based estimate of diffusivity. Results for the Intercomparison monitoring bores are summarised in Tables 2 and 3.

Tidal efficiency and lag are calculated from FFT’s of the ocean tide data and groundwater level responses in the monitoring bores. At each bore

\[
TE_{K1} = \frac{\Phi(v_{K1})_{bore}}{\Phi(v_{K1})_{ocean}} \quad TE_{O1} = \frac{\Phi(v_{O1})_{bore}}{\Phi(v_{O1})_{ocean}}
\]

(13)

and

\[
lag_{K1} = \text{arg} \left( \frac{\Phi(v_{K1})_{bore}}{\Phi(v_{K1})_{ocean}} \right) \quad lag_{O1} = \text{arg} \left( \frac{\Phi(v_{O1})_{bore}}{\Phi(v_{O1})_{ocean}} \right)
\]

(14)

Once tidal efficiency and lag are determined, diffusivity is calculated using equations (7) and (8).

The validity of applying the ‘Ferris model’ to the Cockburn Sound data set is tested by plotting normalised lag verses tidal efficiency on a semi-log graph (Figure 15). The data are compared with each other and the theoretical relationship from equation (9). Most notably, all the bores diverge from the theoretical relationship between tidal efficiency and normalised lag that is predicted by the Ferris model, and there is a clear separation of the deep and shallow bores.

Shallow Bores

There is evidence of a logarithmic relationship between tidal efficiency and normalised lag in the shallow bores but the slope is clearly different to that of the Ferris model (−1/2.π). In general, the observed attenuation of tidal amplitude is greater than the model-predicted attenuation, for the same speed of propagation through the aquifer. Correspondingly, the
observed speed of propagation is faster than the model-predicted speed of propagation, for
the same attenuation of amplitude. This difference is most likely a consequence of the
modelling assumptions and results in estimates of lag-based diffusivity that are consistently
larger than efficiency-based estimates.

Despite this deficiency of the model, the estimates of diffusivity determined from the shallow
bores fall within the range 3,600 to 11,800 m².day⁻¹, which is within a factor of 3.3. Assuming
that the saturated depth of the sand units overlying the Tamala Limestone is around 17
metres (based on the drilling data) and assuming a specific yield of approximately 0.25, the
estimated range in hydraulic conductivity of the sands is 53 to 174 m.day⁻¹, which is
characteristic of medium to coarse sand (Fetter, 1994; Bouwer, 1978).

**Deep Bores**

Figure 15 indicates that the relationship between tidal efficiency and normalised lag in the
deep bores is significantly different compared to both the shallow bores and the Ferris
model. Again, this is likely to be a consequence of the model assumptions, which do not
account for the high-transmissivity Tamala Limestone underlying the sand. The assumptions
of 1-D flow and a homogeneous aquifer, in particular, are poor representations of the true
field conditions.

During the SCOR/LOICZ SGD Intercomparison it was observed that some of the submarine
discharge being measured by benthic flux meters on the seabed was in fact seawater. This is
consistent with the fact that the ocean level was falling during the experiment and indicates
that salt water, previously forced into the aquifer during a period of higher sea level, was
draining back out of the aquifer. The period of the intercomparison corresponds to the first
ten days in Figures 8 to 14 in which the trend of falling sea level is clearly depicted. It is
believed that there is very active exchange of seawater between the ocean and Tamala
Limestone, which is manifest as ‘tidal pumping’.

The current hypothesis is that during periods of rising sea level, saltwater flows into the
Tamala Limestone, which has the effect of displacing both the saltwater interface and free
surface in the overlying sands upward. Downward displacement of the free surface and
saline interface occurs during periods of falling sea level. In effect, rapid propagation of tidal
oscillations through the limestone extends the ocean boundary inshore and underneath of
the freshwater flow system. The fact that the saltwater wedge extends more than two
kilometres inland from the coast is an indication of how transmissive the limestone is
(Section D). In addition, groundwater levels several hundred metres inland from the coast are strongly correlated with monthly average sea level (PPK, 2000).

Because the deep bores are screened just above the limestone, this affects their tidal responses. Compared with the Ferris model, the attenuation of tidal amplitude in the deep bores is much larger than expected for the same speed of propagation. Therefore, the lag-based estimates of diffusivity, which are in the range 8,400 to 28,100 m$^2$.day$^{-1}$, are much larger than the efficiency-based estimates, which are in the range 51,800 to 441,400 m$^2$.day$^{-1}$. For the reasons discussed, however, none of these values are considered to be realistic and it is concluded that the Ferris model is not suited to the particular field conditions.
Table 2: Summary of tidal analysis for K1 tidal constituent

<table>
<thead>
<tr>
<th>Bore ID</th>
<th>Distance to Shore (m)</th>
<th>$TE_{K1}$</th>
<th>Efficiency-based Diffusivity (m²/day)</th>
<th>Lag$_{K1}$</th>
<th>Lag-based Diffusivity (m²/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH1a</td>
<td>179</td>
<td>0.152</td>
<td>27,300</td>
<td>0.094</td>
<td>296,900</td>
</tr>
<tr>
<td>NH2a</td>
<td>93</td>
<td>0.171</td>
<td>8,400</td>
<td>0.117</td>
<td>51,800</td>
</tr>
<tr>
<td>NH2b</td>
<td>96</td>
<td>0.076</td>
<td>4,200</td>
<td>0.293</td>
<td>8,800</td>
</tr>
<tr>
<td>NH3</td>
<td>45</td>
<td>0.301</td>
<td>4,200</td>
<td>0.146</td>
<td>7,800</td>
</tr>
<tr>
<td>NH4</td>
<td>28</td>
<td>0.451</td>
<td>3,600</td>
<td>0.090</td>
<td>7,700</td>
</tr>
<tr>
<td>NH5</td>
<td>18</td>
<td>0.611</td>
<td>4,000</td>
<td>0.064</td>
<td>6,500</td>
</tr>
</tbody>
</table>

Table 3: Summary of tidal analysis for O1 tidal constituent

<table>
<thead>
<tr>
<th>Bore ID</th>
<th>Distance to Shore (m)</th>
<th>$TE_{O1}$</th>
<th>Efficiency-based Diffusivity (m²/day)</th>
<th>Lag$_{O1}$</th>
<th>Lag-based Diffusivity (m²/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH1a</td>
<td>179</td>
<td>0.147</td>
<td>28,100</td>
<td>0.075</td>
<td>441,400</td>
</tr>
<tr>
<td>NH2a</td>
<td>93</td>
<td>0.174</td>
<td>9,100</td>
<td>0.044</td>
<td>347,600</td>
</tr>
<tr>
<td>NH2b</td>
<td>96</td>
<td>0.075</td>
<td>4,400</td>
<td>0.246</td>
<td>11,700</td>
</tr>
<tr>
<td>NH3</td>
<td>45</td>
<td>0.321</td>
<td>5,000</td>
<td>0.115</td>
<td>11,800</td>
</tr>
<tr>
<td>NH4</td>
<td>28</td>
<td>0.493</td>
<td>4,900</td>
<td>0.081</td>
<td>9,000</td>
</tr>
<tr>
<td>NH5</td>
<td>18</td>
<td>0.639</td>
<td>5,200</td>
<td>0.050</td>
<td>10,200</td>
</tr>
</tbody>
</table>
Figure 15: Non-dimensional plot of normalised lag verses tidal efficiency for groundwater monitoring bores NH1 to NH5
ACKNOWLEDGEMENTS

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The Maritime Division of Department of Transport, Western Australia, supplied Fremantle tide data free of charge.
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